Sendai Logic School 2018

Tohoku University and Akiu Spa Hotel Iwanumaya, Sendai, Japan

7-9 December, 2018

Contents

Organizers

Kazuyuki Tanaka (Tohoku University)

Takayuki Kihara (Nagoya University)

NingNing Peng (Wuhan University of Technology)

Part I

Program

Program1: Tutorials

- December 7 at Tohoku University

Session 1: chaired by Kazuyuki Tanaka

14:30 - 15:00	Reception, Opening
15:00 - 16:00	Toshimichi Usuba "Syntactical and semantical approaches to generic multiverse"
16:15 - 17:15	Yue Yang "Ramsey typed theorems and reverse mathematics"
18:00 - 20:00	(banquet)

- December 8 at Hotel Iwanumaya

Session 2: chaired by Takayuki Kihara

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15:40 - 16:40	Daisuke Ikegami "On preserving AD via forcings"
16:50 - 17:50	Longyun Ding "Reductions among analytic and co-analytic equivalence relations"
18:30 - 20:30	(banquet)
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- December 9 at Hotel Iwanumaya

Session 3: chaired by Peng NingNing

10:00 - 11:00	Kenshi Miyabe "Construction of random and non-random sets"
11:10 - 12:10	Guohua Wu "On the degree spectra of orderings on computable torsion-free abelian groups"
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Program2: Short Communications

Session 4: chaired by Peng Weiguang

14:00 - 14:30	Takayuki Kihara "BQO-Wadge theory on ultrametric spaces"
14:40 - 15:10	Wenjuan Li "One modal μ-calculus Part1"
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20:00 - 23:00	free discussion time

Part II
Abstracts

1 Syntactical and semantical approaches to generic multiverse

Toshimichi Usuba (Waseda University)

Keywords: generic multiverse, forcing, generic extension, ground model.

The concept of generic multiverse was first introduced by Woodin. Generic multiverse is a collection of transitive models of ZFC which is closed under taking ground models and generic extensions. Later Steel gave a similar but different definition of generic multiverse, and Väänänen developed some syntactical formal system of Steel's generic multiverse. On the other hand, Steel's definition is incompatible with Woodin's one, and Väänänen's system cannot treat Woodin's multiverse by some reasons. In this talk, we discuss other syntactical and semantical framework of generic multiverse which can treat Woodin's and Steel's one simultaneously.

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2 Ramsey typed theorems and reverse mathematics

Yang Yue (National University of Singapore)

RAMSEY TYPED THEOREMS AND REVERSE MATHEMATICS

YANG YUE

Abstract. The object of this talk is to introduce Reverse Mathematics to graduate students, with an emphasis on nonstandard models. I will give a brief survey about the previous works in this area, using Ramsey's Theorem as the main example. I will also talk about a recent joint work with Chitat Chong, Li Wei and Wang Wei on Ramsey's Theorem for trees.

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3 Bounded Arithmetic and forcing

Satoru Kuroda (Gunma Prefectural Women's University)

Keywords: Bounded Arithmetic, Complexity Theory, Forcing.

In this talk, I will survey basic notions and results in bounded arithmetic. Specifically, two-sort theories a la Cook and Nguyen for PTIME and its subclasses and their relations with propositional proof systems are considered. Then two types of forcing constructions of models of bounded arithmetic are discussed, namely Krajicek forcing and Takeuti-Yasumoto forcing.

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4 On preserving AD via forcings

Daisuke Ikegami (Shibaura Institute of Technology)

Keywords: Set theory, Descriptive set theory, The Axiom of Determinacy, Forcing, Large cardinals.

It is known that many concrete forcings such as Cohen forcing destroy AD. In this talk, we show that one cannot preserve AD via forcings as long as the forcing increases Θ and V satisfies AD^+ and V=L(P(R)). We also provide an example of forcings which preserve AD while increasing Θ when V is not of the form L(P(R)). This is joint work with Nam Trang.

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5 Reductions among analytic and co-analytic equivalence relations

Longyun Ding (Nankai University)

Keywords: equivalence relations, Borel reduction, analytic sets, co-analytic set.

Borel reduction is a very powerful tool for studying Borel equivalence relations and orbit equivalence relations. However, for more general equivalence relations, Borel reduction is not strong enough. This forces us to consider more different reductions. In this talk, we review some main results on Borel reduction and introduce some new reductions on analytic and co-analytic equivalence relations.

6 Construction of random and non-random sets

Kenshi Miyabe (Meiji University)

Keywords: Schnorr randomness, computable randomness, high.

CONSTRUCTION OF RANDOM AND NON-RANDOM SETS

KENSHI MIYABE

The theory of algorithmic randomness defines many notions of randomness. In particular, we have the following inclusions:

$$\mathrm{SR} \supsetneq \mathrm{CR} \supsetneq \mathrm{MLR}$$

where SR, CR, MLR are the classes of Schnorr random sets, computablly random sets, and Martin-Löf random sets, respectively. Their separations are basic and important results, but the proof is somewhat complicated, especially between SR and CR.

The main goal of the talk is to explain the proof idea of the following theorem.

Theorem 1 (Nies, Stephan, and Terwijn [1]). *The following are equivalent.*

(i) *A is high.*
(ii) *There is a set $B \equiv_T A$ that is computably random but not ML-random.*
(iii) *There is a set $C \equiv_T A$ that is Schnorr random but not computably random.*

The main tool is a martingale. Many notions of randomness are characterized by boundedness of martingales in some classes. It is often the case that we prove the proper inclusion between two randomness notions by constructing a set that is random in the sense that all martingales in a class are bounded (or growing slowly) along the set, and that is non-random in the sense that some martingale is unbounded (or growing fast).

We can also discuss the Turing degrees that are necessary to construct such sets.

References

[1] A. Nies, F. Stephan, and S. Terwijn. Randomness, relativization and Turing degrees. *Journal of Symbolic Logic*, 70:515–535, 2005.

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7 On the degree spectra of orderings on computable torsion-free abelian groups

Guohua Wu (Nanyang Technological University)

Keywords: Computable torsion-free abelian groups, orderings, Degree spectra.

In this talk, I will present a joint work with Frank Stephan (NUS) and Huishan Wu (BLCU), on the Turing degrees of orderings on computable torsion-free abelian groups. Our work is motivated by a paper of Kach, Lange and Solomon (APAL 2013), which shows the existence of a c.e. set C and computable torsion-free abelian group G with infinite rank, admitting exactly two computable orderings such that every C-computable order on G is computable. In this paper, Kach, et al. pointed out that there are infinitely many such Cs, and these Cs can have low degree. Martin shows in his PhD thesis (Wisconsin) that C can be of high degree. Our main result provides a close relation between such Cs and PA degrees, and shows that such Cs can be any incomplete c.e. set.

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8 BQO-Wadge theory on ultrametric spaces

Takayuki Kihara (Nagoya University)

Keywords: Better quasi order, Wadge degree, descriptive set theory, computability theory, Borel determinacy.

Recently, Kihara and Montalban [1] have given a complete classification of the BQO-valued Borel maps on Baire space in terms of the Wadge degrees. Roughly speaking, the authors have shown that any BQO-valued Borel map on Baire space is generated by (suitable iterations of) very simple construction principles, and is completely classified by detecting how it is constructed. In this talk, we extend the theory of Wadge degrees on BQO-valued maps to arbitrary (possibly non-separable) completely ultrametrizable spaces. We apply suitable variants of computability theory over arbitrary uncountable cardinals to generalize Kihara-Montalban's complete classification of the BQO-valued Borel maps to the non-separable setting.

[1] Takayuki Kihara and Antonio Montalban, On the structure of the Wadge degrees of BQO-valued Borel functions, to appear in Transactions of the American Mathematical Society.

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9 One modal μ-calculus Part 1

Wenjuan Li (Nanyang Technological University)

Keywords: Modal μ-calculus, Alternation hierarchy.

Modal μ-calculus is an extension of modal logic by adding greatest and least fixpoint operators, which is closed related with tree automata and parity games. It is natural to classify the formulas of modal μ-calculus by the number of alternating blocks of fix point operators, which is called the alternation hierarchy. At the same time, the number of variables contained in a formula also serves as an important measure of complexity for formulas of modal *mu*-calculus. A fundamental issue is on the strictness of such alternation hierarchies for (fragments of) modal μ-calculus.

In the first half of this talk, we review the historical and latest studies on the strictness of alternation hierarchy, as well as the variable hierarchy of modal *mu*-calculus.

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10 One modal μ-calculus Part 2

Misato Nakabayashi (Tohoku University)

Keywords: One-variable modal μ-calculus, Weak alternating tree automaton, Weak parity game, Transfinite extension.

Modal μ-calculus, introduced by Kozen, is an extension of modal propositional logic by adding a greatest fixpoint operator μ and a least fixpoint operator ν. It is well suited for specifying properties of transition systems, and closely related to tree automata and parity games. First, we show the relationship between one-variable L_μ-formulas and weak alternating tree automata. Furthermore, the alternation depth of a one-variable L_μ-formula φ corresponds to the number of the priorities of the associated automaton $\mathcal{A}$. Therefore, we conclude the strictness of the alternation hierarchy of one-variable L_μ-formulas within Δ_2^μ from the strictness of the hierarchy of weak alternating tree automata first proved by Mostowski. Second, we will give a simpler proof of its strictness. Finally, we introduce weak parity games with infinitely many priorities, whose winning positions can be expressed by a Δ_2^μ formula, but requires a transfinite extension of one-variable L_μ-formulas.

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11 Solovay reduction and continuity

Toshio Suzuki (Tokyo Metropolitan University)

Keywords: Solovay reduction, left c.e. reals, Lipschitz continuity, Hoelder continuity, partial randomness.

This work is collaboration with Yuki Mizusawa and Masahiro Kumabe. Solovay reduction is a stronger form of Turing reduction based on distance of two reals. As far as left c.e. reals, the completeness with respect to Solovay reduction coincides with 1-randomness. In this talk, we would like to understand Solovay reduction in terms of continuity. We observe that Solovay reduction can be characterized as existence of a certain Lipschitz function that is computable in the sense of Weihrauch's book (2000). By the way, in a closed interval of the real line, some continuity concepts have an order of implication. Hoelder continuity with positive exponent ¡ 1 is known as a slightly weaker form of Lipschitz continuity. We introduce a slightly weaker form of Solovay reduction, a pseudo Solovay reduction. We separate Solovay reduction, pseudo Solovay reduction and Turing reduction. We show some examples of complete sets with respect to pseudo Solovay reduction that are partial random in the sense of Tadaki (2002). Finally, we show the relationships between Hoelder continuity and pseudo Solovay reduction.

Part III

Bus schedule

Sendai Seibu Air port Liner (For Sendai airport and Sendai station)

Akiu-onsen-yumoto	Sendai station	Sendai airport
06:57	07:35	08:10
09:02	09:40	10:15
10:17	10:55	11:30
12:12	(Don't stop)	13:01
13:57	14:35	15:10
17:02	17:40	18:15

Sendai Seibu Liner (For Sendai station)

Sakan-mae	Sendai station
06:56	07:27
08:26	08:59
09:32	10:04
10:32	11:04
11:32	12:04
12:26	12:58
13:26	13:58
14:32	15:04
15:32	16:05
16:26	16:59
17:26	17:59
18:21	18:54
19:21	19:54

Miyakoh-bus (For Sendai station)